THE NUCLEAR-MATTER RESPONSE IN THE QUARK STRING-FLIP MODEL

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Nuclear matter is modeled directly in terms of its constituent quarks. A many-body string-flip potential is used that confines quarks within hadrons, enables the hadrons to separate without generating van der Waals forces, and is symmetric in all quark coordinates. We present variational Monte Carlo results for the ground-state properties of large, three-dimensional systems. A phase transition from nuclear to quark matter is observed which is characterized by a dramatic rearrangement of strings. We report on exact calculations of the dynamic response of many-quark systems in one spatial dimension. At low density and small momentum transfers the response is substantially larger than that of a free Fermi gas of quarks; this suggests that there is a coherent response from all the quarks inside the hadron. This coherence, however, is incomplete, as the response is suppressed relative to that of a free Fermi gas of nucleons due to the internal quark substructure of the hadron.

1 Introduction

The advent of new, high-energy facilities for nuclear physics research offers a unique possibility at identifying departures from a conventional, meson-baryon description of nuclear phenomena. Naively, one would expect that as the typical distance scales probed become short relative to the nucleon size, quarks and gluons should become manifest in nuclear observables. Yet, quark-gluon signatures in nuclear phenomena have, so far, proved elusive. One of the objections most often raised by the critics of conventional nuclear structure models is that, because the intrinsic size of the hadrons, a picture of nucleons interacting in the medium via meson exchanges is inappropriate. Yet, there seems to be ample experimental evidence that will support that, although some properties of the nucleon may be modified in the medium, a nucleon inside the medium resembles to a very good approximation a nucleon in free space. Perhaps one of the greatest challenges facing nuclear physics today is the explanation of these remarkable facts: why are these effective hadronic models so successful and how can such models emerge from the basic underlying theory having quarks and gluons as the fundamental degrees of freedom. It is also interesting to note that, although most of these questions have been posed since the advent of QCD, little progress has been made in answering them. A serious difficulty encountered in attempting to answer these questions is how to model a system that is believed to have quarks confined inside color-neutral hadrons at low density but free quarks at high density. The divorce of the two pictures is perhaps forced by the difficulty of

treating quark confinement: how can quarks be confined inside hadrons, yet hadrons can separate without generating long-range van der Waals forces? We offer no new insights into this difficult problem; rather we argue that it is useful to consider an effective model which interpolates between a hadron- and quark-based description at low and high density.

In this contribution we consider nuclear matter from the viewpoint of the constituent quark model. The motivation for this study is threefold. First, we wish to examine the role of nucleon substructure in nuclear observables. For example, we wish to understand how are hadronic properties—such as the nucleon form factor—modified in the nuclear environment. Further, we wish to understand, as a function of the density and the momentum transfer, when do leptons scatter from nucleons and when do they scatter from individual quarks. Second, we want to identify signatures for the nuclear- to quark-matter transition. For example, how is the color susceptibility modified as a function of the density of the system. Finally, we want to search for qualitatively new modes of excitation in many-quark systems. These modes—which could be collective excitations of several quarks in the nucleus—are not present in single hadrons nor in hadronic models of the nucleus. These "quark giant resonances" could involve the coherent response of many quarks to density, spin, flavor, or color fluctuations. Moreover, we are interested in studying the mixing between quark-like excitations, such as the $N \to \Delta$ transition, and hadronic excitations, such as the Gamow-Teller resonance. Indeed, it is unknown how the excitation energy and mixing of these modes will change with density.

2 The String-flip Model

The string-flip, or quark-exchange, model[1] is a simple many-body generalization of the nonrelativistic constituent quark model[2]. Yet the obvious generalization consisting of pairwise confining forces between quarks is known to generate long-range van der Waals forces[3]. These long-range forces which are power-law—rather than exponentially—suppressed at long distances do not exist in nature. Rather, it is cluster separability, namely, the possibility for color-singlet hadrons to separate without generating residual van der Waals forces, that is observed in nature. The string-flip model succeeds in providing cluster separability due to the intrinsic many-body nature of the potential[4,5,6,7,8,9].

The Hamiltonian for the present version of the string-flip model is given by[7,8]

$$H = \sum_{i=1}^{N} \frac{P_i^2}{2m} + V(\mathbf{r}_1, \ldots, \mathbf{r}_N) \; . \tag{1}$$

We consider equal numbers ($A = N/3$) of red, blue, and green quarks; for simplicity we assume that quarks are devoid of any additional (spin and flavor) intrinsic degree

of freedom. We require each color-singlet hadron to be formed by one red, one blue, and one green quark. Moreover, we require the grouping of quarks into color-singlet hadrons to be optimal. That is (we use units in which $k = m = \hbar = 1$)

$$V(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \mathrm{Min} \sum_{i=1}^{A} \frac{1}{2} \left[(\mathbf{r}_i^R - \mathbf{r}_i^B)^2 + (\mathbf{r}_i^B - \mathbf{r}_i^G)^2 + (\mathbf{r}_i^G - \mathbf{r}_i^R)^2 \right] . \tag{2}$$

Here $\mathbf{r}_i^R$, $\mathbf{r}_i^B$, and $\mathbf{r}_i^G$, are the positions of the red, blue, and green quarks "belonging" to the i^{th} hadron, and the minimization procedure is taken over a—potentially enormous—set of $(A!)^2$ possible groupings. An example of an optimal and non-optimal grouping of six quarks into two color-singlet hadrons is depicted in Fig. 1. The many-body potential represents an adiabatic approximation to the extremely

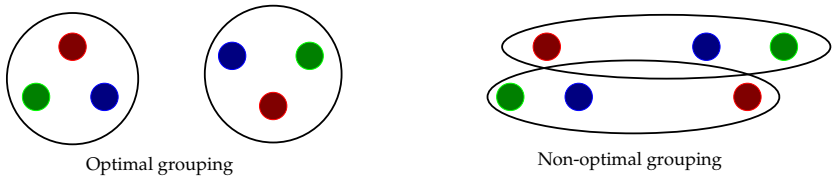

Figure 1. Optimal and non-optimal grouping of quarks into hadrons.

complicated quark-gluon dynamics. We assume that the gluons are "light" degrees of freedom with a dynamical time-scale that is much shorter than that of the "heavy" quarks; as quarks move, the gluonic strings adjust instantaneously to the position of the quarks. The basic problem, then, is to decide how quarks are to be grouped inside the hadrons: a gluonic string (or flux tube) leaving a quark must end up in a three-quark junction (see Fig. 2). The fundamental question becomes which junction? Presumably, lattice QCD solves this problem—but at a spectacular computational cost. It is because of these enormous computational demands that one must resort to simple phenomenological models. Nevertheless, most models of the many-quark dynamics will need to determine which quarks belong to a particular hadron. Thus, solving some kind of "quark-assignment" problem is likely to be a general requirement for these models. The potential is a many-body operator; it can not be reduced to a sum of two-body terms. Indeed, the movement of a single quark might cause several string to flip. Moreover, the many-body potential is symmetric under the exchange of all the quark coordinates—even those "belonging" to different hadrons. Quarks are also confined inside color-singlet hadrons. However, there are now no long-range van der Waals forces between the hadrons, as the force now saturates within the hadron. Thus, the only residual interaction between hadrons involves the possibility of quark exchange and the Pauli principle between identical quarks. This immediately suggests that a typical distance scale characterizing the interaction is determined by the

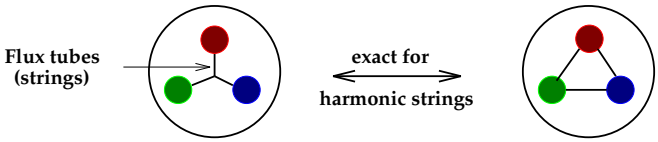

Figure 2. Adiabatic approximation to the complicated quark-gluon dynamics.

size of the hadron; if the distance between hadrons is much larger than this size, quark-exchange should be suppressed. Any long-range component to the hadron-hadron potential, arising phenomenologically from the exchange of pseudo-Goldstone bosons, is beyond the scope of this simple model.

3 Static Properties

We compute the ground-state properties of the many-quark system using a variational Monte Carlo approach. The dynamics of the system is contained in a simple one-parameter variational wave function of the form[7,8]:

$$\Phi_\lambda(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \exp\,[\,-\lambda V(\mathbf{r}_1,\ldots,\mathbf{r}_N)]\Phi_{\rm FG}(\mathbf{r}_1,\ldots,\mathbf{r}_N)\;. \tag{3}$$

The Fermi-gas wave function ($\Phi_{\rm FG}$) is a product of red, blue, and green Slater determinants. The Fermi-gas wave function is exact for a system of identical fermions with no correlations other than those generated by the Pauli exclusion principle. The exponential factor, on the other hand, characterizes the amount of clustering in the ground state through the variational parameter λ. For a dilute system of quarks—where quarks cluster into individual color-singlet hadrons—the variational wave function reproduces the exact wave function of isolated clusters in the limit of $\lambda \to 1/\sqrt{3}$. Moreover, in the high-density limit—where the interparticle separation is substantially smaller than the confinement scale—the potential energy becomes unimportant and the variational wave function reproduces the exact Fermi-gas result in the limit of $\lambda \to 0$. Thus, this simple one-parameter variational wave function is exact in the low- and high-density limits, with $\lambda^{-1/2}$ playing the role of a confinement scale. One of the advantages of using such a simple variational wave function is that the expectation value of the kinetic and potential energies are not independent:

$$\langle\Phi_\lambda|T|\Phi_\lambda\rangle = T_{\rm FG} + 3\lambda^2\langle\Phi_\lambda|V|\Phi_\lambda\rangle\;. \tag{4}$$

Here $T_{\rm FG}$ is the kinetic energy of a free Fermi gas and the additional term, $3\lambda^2\langle V\rangle$, represents the increase in kinetic energy above the Fermi-gas limit due to the presence of clustering correlations. Thus, to compute the total energy of the system we only

need to evaluate the expectation value of the potential energy

$$E_\lambda(\rho) \equiv \langle \Phi_\lambda | H | \Phi_\lambda \rangle = T_{\rm FG} + (3\lambda^2 + 1)\langle \Phi_\lambda | V | \Phi_\lambda \rangle \ . \tag{5}$$

This expectation value is relatively simple to compute using Metropolis Monte Carlo methods. Yet, one must realize that it involves performing a $3N \simeq 300$-dimensional integral!

The most demanding component of the calculation is the determination of the optimal grouping of quarks into color-singlet hadrons. Indeed, for the potential of Eq. 2 there is no efficient (i.e., power-law) algorithm to solve this complicated assignment problem. Thus, we have resorted to the stochastic optimization technique of simulated annealing[8]. We have computed the energy per quark and length-scale for quark confinement as a function of density. Although these results already show how quark clustering decreases with density and characterize the nuclear- to quark-matter transition, they are limited to a very small ($A = 8$) number of hadrons. Thus, we find it advantageous to study a slightly different model but one in which the assignment problem can be solved readily. For this problem we consider pairing two colors at a time[7]. That is,

$$V(\mathbf{r}_1, \ldots, \mathbf{r}_N) = {\rm Min} \sum_{i=1}^{A} \frac{1}{2}(\mathbf{r}_i^R - \mathbf{r}_i^B)^2 + {\rm Min} \sum_{i=1}^{A} \frac{1}{2}(\mathbf{r}_i^B - \mathbf{r}_i^G)^2 + {\rm Min} \sum_{i=1}^{A} \frac{1}{2}(\mathbf{r}_i^G - \mathbf{r}_i^R)^2 \ . \tag{6}$$

Note that in this case one searches for the optimal pairing of red and blue quarks—independent of the position of the green quarks; one then repeats the procedure to find the corresponding optimal blue-green and green-red pairings. An exhaustive approach to the pairing problem requires of a search among $A!$ different configurations. However, efficient pairing algorithms—with a computational cost proportional to N^3—have been already developed by mathematicians[10] and implemented by economists; economists have long been interested in the problem of pairing N factories with N retail stores in order to minimize the overall cost of exchanging goods. In spite of the change, both models share many common features. Clearly, they are identical in the case of an isolated cluster, and, thus, also in the very-low density limit. Moreover, both models guarantee cluster separability, thus avoiding the emergence of van der Waals forces. However, there are some differences. Most notoriously, by pairing quarks independently there is no guarantee that the quarks will be grouped into three-quark clusters; color-neutral hadrons in this model may contain any multipole of three quarks (see Fig 3). We have computed the energy per quark and variational parameter as a function of density for a total of $N = 96$ quarks (no figure is shown). We have identified a discontinuity in the derivative of the energy and an abrupt transition in the variational parameter[7]. This transition from nuclear- to quark-matter is accompanied by a dramatic change in the quark pairings. To characterize this transition we define an n-quark cluster probability (with n any

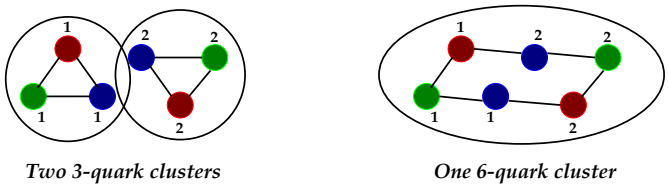

Figure 3. Differences between the models due to possible multi-quark configurations.

multipole of 3) as the probability of finding a quark as part of an n-quark cluster. In Fig. 4 we display the n-quark cluster probabilities as a function of density. In the low-density nuclear phase most of the quarks are clustered into simple nucleons containing three quarks each. In contrast, the 3-quark cluster probability drops to about one third in the quark-matter phase. The 6-, 9-, and 12-quark cluster probabilities are small at all densities. The majority of the probability resides in large clusters involving 15 or more quarks. Indeed, the size of these clusters are comparable to the simulation volume. This transition—in which the strings grow and fill the simulation volume—is analogous to a percolation phase transition observed in some condensed-matter systems. Clearly, our cluster probabilities are model dependent. Indeed, they depend sensitively on the definition of the many-quark potential [see Eq. (6)]. Yet, these quark-cluster probabilities were predicted within our model; this is in contrast to the majority of theoretical approaches which treat multi-quark cluster probabilities as arbitrary parameters.

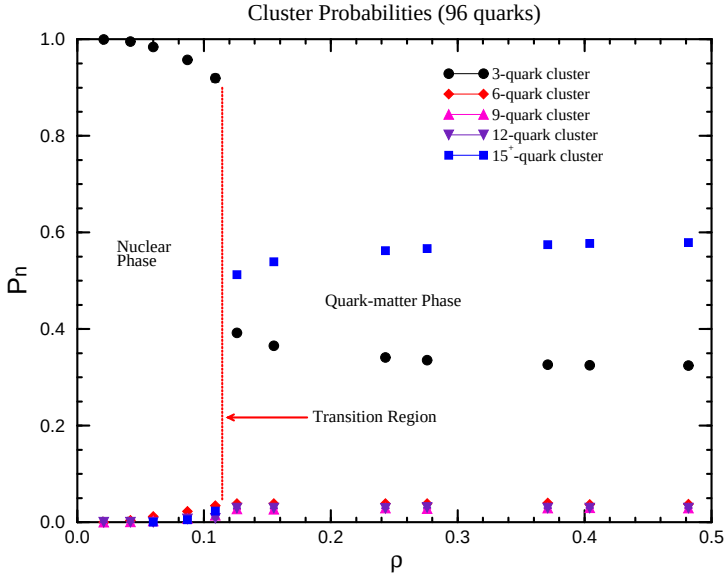

Figure 4. Quark-cluster probabilities as a function of density.

4 Dynamic Properties

In this section we report on our progress towards calculating the exact dynamic response of hadronic matter. To our knowledge, this is the first time that an attempt has been made at modeling the response of a many-body system with confined degrees of freedom. We are interested in computing the longitudinal (or charge) response of the system,

$$S_L(\mathbf{q},\omega) = \sum_n \Big|\langle\Psi_n|\hat{\rho}(\mathbf{q})|\Psi_0\rangle\Big|^2 \delta\Big[\omega - (E_n - E_0)\Big] \ . \tag{7}$$

Here $\hat{\rho}$ is the charge-density operator, Ψ_0 is the exact ground state of the system, and Ψ_n is an excited state with excitation energy $E_n - E_0$. Note that the response is probed at a momentum transfer $\mathbf{q}$ and at an energy loss ω. To simulate the response it is convenient to introduce its Fourier transform—the autocorrelation function:

$$S_L(\mathbf{q},t) = \int_0^\infty d\omega e^{-i\omega t} S_L(\mathbf{q},\omega) = \langle\Psi_0|\hat{\rho}^\dagger(\mathbf{q}) e^{-i(\hat{H}-E_0)t}\hat{\rho}(\mathbf{q})|\Psi_0\rangle \ . \tag{8}$$

This form is suitable to be simulated in the computer, but only in Euclidean (or imaginary) time: $t \to -i\tau$; the main difficulty in simulating the response in real time originates from its oscillating phase. It is interesting to note that the same imaginary-time evolution operator that appears in the calculation of the response can be used to compute the exact ground state of the system[11]

$$\lim_{\tau\to\infty} e^{-(\hat{H}-E_0)\tau}|\Phi\rangle = |\Psi_0\rangle\langle\Psi_0|\Phi\rangle \ . \tag{9}$$

Thus, the Euclidean response of the system can be calculated exactly, up to statistical uncertainties. However, two serious challenges remain to be addressed: a) how to analytically continue the Euclidean response to real time and b) how to deal with the large statistical uncertainties that are ubiquitous to the simulation of fermionic systems. The first problem emerges from the realization that the inverse Laplace transform of numerical data represents an ill-posed problem. A promising tool in the reconstruction of the real-time response is the method of maximum entropy[12]. Maximum entropy uses a Bayesian approach to the problem; it finds the most probable real-time response, $S_L(\mathbf{q},\omega)$, that has the computed Euclidean response as its Laplace transform. In essence, maximum entropy represents a sophisticated chi-square minimization procedure—aided by physics (in the form of a model and sum rules) and assumptions about smoothness and positivity. The second problem represents a formidable challenge, indeed. The essential difficulty arises from cancelling (plus and minus) signs associated with fermionic determinants[13]. Although useful strategies might exist, these cancellations can not be avoided in more than one spatial dimension. In the special case of one dimension the sign cancellation problem can be circumvented by working in an ordered subspace in which the wave function can be chosen to be positive definite; as a quark attempts to move pass one of its neighbors,

the emergence of a large and repulsive "Pauli potential" precludes the move. Hence, once ordered, the quarks will remain order throughout the simulation. Because of the formidable challenges encountered by exact three-dimensional calculations, we concentrate for the rest of this contribution on the exact simulation of many-quark systems in one spatial dimension. Moreover, we will also avoid discussing the analytic continuation of the Euclidean response. Thus, in this contribution we focus exclusively on the Coulomb sum $S_L(q, t = 0)$.

For the simulations we employ the original string-flip model developed by Lenz and collaborators[1]. Here, quarks are restricted to one spatial dimension and are devoid of any intrinsic degrees of freedom. Moreover, quarks are confined within two-quark hadrons by harmonic forces. In spite of its simplicity, the model shows rich and interesting behavior associated with the internal quark substructure of the hadron[4,6]. It is the impact of this quark substructure on the response that we now address. In Fig. 5 we display the Coulomb sum as a function of the momentum transfer

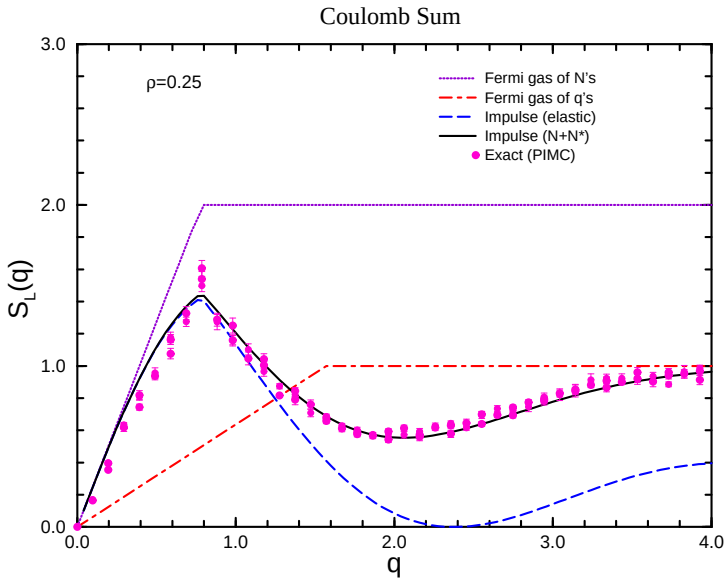

Figure 5. Coulomb sum as a function of momentum transfer.

for a density of $\rho = 0.25$. For this density the confinement scale is considerably smaller than the average inter-hadron separation; thus a nucleon at this density—although slightly "swollen"—resembles to a very good approximation a nucleon in free space. The dot-dashed line represents the response of free Fermi gas of quarks. It is evident that for small q the Fermi-gas response is well below the exact response of the system. This behavior suggests that at these momentum transfers both quarks in a hadron respond coherently. However, the coherence is incomplete, as the response of the system is below that of a free Fermi gas of nucleons (dotted line). Some of the coherence is lost because of the intrinsic quark substructure of the nucleon; the

nucleon "survival" probability is proportional to its elastic form factor. Hadronic descriptions of the nuclear response rely on the impulse approximation; one assumes that the probe couples to a point-nucleon current multiplied by the elastic form factor of the nucleon—which remains unchanged from its free-space value. The long-dashed line depicts the outcome of such a calculation. Note that here we have assumed that the point-nucleon response is that of a free Fermi gas of nucleons. The impulse approximation gives, indeed, an excellent description of the numerical data for small momentum transfers. However, as the momentum transfer becomes comparable to the inverse nucleon size, there is substantial strength in the exact response that the simple impulse-approximation approach can not account for. This extra strength must be contained in the nucleon resonances. To test such a scenario we introduce an extended impulse-approximation approach that incorporates all nucleon resonances. In this limit, the dynamic response of the system is proportional to the product of the Fermi-gas response of "point" nucleons times the single-nucleon response:

$$\frac{1}{N}S_{EIA}(q,\tau) = \frac{1}{2}e_N^2\left(\frac{S_A^{FG}(q,\tau)}{A}\right)\sum_{n=0}^{\infty} e^{-2n\omega_0\tau}\left|F_{n0}(q)\right|^2 . \tag{10}$$

Here $F_{n0}(q)$ is a single-nucleon transition form factor, ω_0 is the oscillator frequency, and $e_N^2 = 4$ is the square of the nucleon charge (the quark charge has been defined to be one). Note that some of the dimensionful parameters have been temporarily restored. The outcome of such a calculation is shown with the solid line. The agreement with the exact numerical calculation is excellent over the entire momentum-transfer range. The widely used impulse approximation, which is obtained from the above expression by retaining only the $n = 0$ term, misses most of the intermediate- and high-q strength. The extra strength is, indeed, contained in the excitation of the nucleon resonances.

5 Conclusions

We have examined the role of the quark substructure of hadrons on nuclear observables using simple constituent quark models. The string-flip models used here confine quarks within hadrons, enable the hadrons to separate without generating van der Waals forces, and are explicitly symmetric in all quark coordinates. The crucial feature of these models is the need to determine an optimal grouping of quarks into hadrons. In a three-dimensional model with a (global) color degree of freedom we have identified a nuclear- to quark-matter transition—characterized by a dramatic rearrangement of strings. We have also calculated the exact Coulomb sum of a many-quark system in one spatial dimension. At low density and small momentum transfers the response was considerably smaller than that of a free Fermi gas of quarks; this suggests a coherent response from all the quarks in the hadron. However, this coherence was incomplete, as the internal quark substructure of the hadron led to a

suppression of the response relative to that of a free Fermi gas of nucleons. As the momentum transfer increased and became comparable to the inverse nucleon size, substantial strength above the impulse-approximation limit was clearly identified. We have concluded that most of this extra strength was contained in the excitation of the nucleon resonances.

In the future, we would like to include additional intrinsic (spin-isospin) degrees of freedom into our simulations. We will also continue our search, via the maximum-entropy method, for quark giant resonances. Further, we could easily add strange quarks into our simulations by assuming a flavor-independent form for the potential; one obtains the optimal grouping of quarks into hadrons irrespective of the flavor of the quarks. This will enable us to study the transition to strange matter. In summary, we have shown that, in spite of their apparent simplicity, string-flip models of nuclear matter display rich behavior that could result in valuable insights into the role of nucleon substructure in hadronic physics.

6 Acknowledgments

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